

# New Approach to Attitude/Momentum Control for the Space Station

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A new approach to control moment gyro (CMG) momentum management and attitude control of the space station is presented. The control algorithm utilizes both the gravity-gradient and gyroscopic torques to seek torque equilibrium attitude in the presence of secular and cyclic disturbances. It is shown that, depending on mission requirements, either pitch attitude or pitch-axis CMG momentum can be held constant. It is also shown that a combination of yaw attitude and roll-axis CMG momentum can be held constant, whereas a combination of roll attitude and yaw-axis CMG momentum cannot. As a result, the overall attitude and CMG momentum oscillations caused by cyclic aerodynamic disturbances are minimized. A state feedback controller with minimal computer storage requirements for gain scheduling is also presented. An inherent physical property of the coupled roll/yaw dynamics is discussed in terms of a transmission zero of a multivariable system.

## Introduction

THE space station shown in Fig. 1 will employ control moment gyros (CMG's) as a primary actuating device during normal flight mode operation. Since the CMG's are momentum exchange devices, external control torques must be used to desaturate the CMG's, i.e. bring the momentum back to nominal value. Some methods for unloading CMG momentum include the use of magnetic torques, reaction jets, and gravity-gradient torque. For the space station, the gravity-gradient torque approach is preferred because it requires no consumables or additional hardware. For this reason, various schemes using gravity-gradient torque have been developed.<sup>1-9</sup>

Two approaches to momentum management using gravity-gradient torque are possible. The first of these is a "discrete" or "periodic" momentum control approach<sup>1-5</sup> that is basically a feedforward open-loop control scheme. This scheme uses appropriate samples of CMG momentum, with the sampling frequency of the same order as the orbital frequency. This approach, as well as the "predictive" approach,<sup>7</sup> can handle expected momentum changes; however, it requires accurate inertia matrix properties and environmental models to generate the proper attitude steering command. In the "discrete" or "periodic" approach, the attitude controller, with a much higher control bandwidth, is designed independently.

The second approach to CMG momentum management is a "continuous approach"<sup>6-9</sup> that integrates the momentum management and attitude control design. In this continuous, closed-loop control of both the CMG momentum and station attitude, the design objective is to establish a proper tradeoff between station pointing and CMG momentum management, while satisfying the specific mission requirements.

Expanding on this "continuous" approach, we present a new scheme to the space station momentum management and attitude control. The proposed controller provides a proper disturbance accommodation. As a result, the cyclic peak of the station attitude and CMG momentum oscillation caused by aerodynamic torque is minimized. In this paper, CMG's are considered as ideal torquers; however, the CMG gimbal dynamics as well as the CMG steering law<sup>10,11</sup> should be included in the further development of an efficient control law for the overall system. Structural flexibility of the space station considered in Refs. 12 and 13 is neglected because of the low bandwidth nature of the integrated momentum/attitude controller. We assume that a strapdown inertial reference system provides relatively noise-free estimates of all states and that the body-axis components of CMG momentum can be measured. Practical multivariable controller synthesis is accomplished employing various techniques: the classical control approaches,<sup>12-17</sup> the linear-quadratic-regulator (LQR) synthesis technique,<sup>18,19</sup> asymptotic disturbance rejection,<sup>19,20</sup> decentralized (partial state feedback) control,<sup>21</sup> and robust eigensystem assignment techniques.<sup>22-24</sup> A brief summary of a pole-placement algorithm for a partial state feedback controller is given in the Appendix.

## Mathematical Models for Controller Synthesis

The space station in circular orbit is expected to maintain local vertical and local horizontal (LVLH) orientation during normal mode operation. The nonlinear equations of motion in terms of components along the body-fixed control axes can be written as

Space station dynamics:

$$\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = - \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \\ + 3n^2 \begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \\ + \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{c}_3 \end{bmatrix} + \begin{bmatrix} -u_1 + w_1 \\ -u_2 + w_2 \\ -u_3 + w_3 \end{bmatrix} \quad (1)$$

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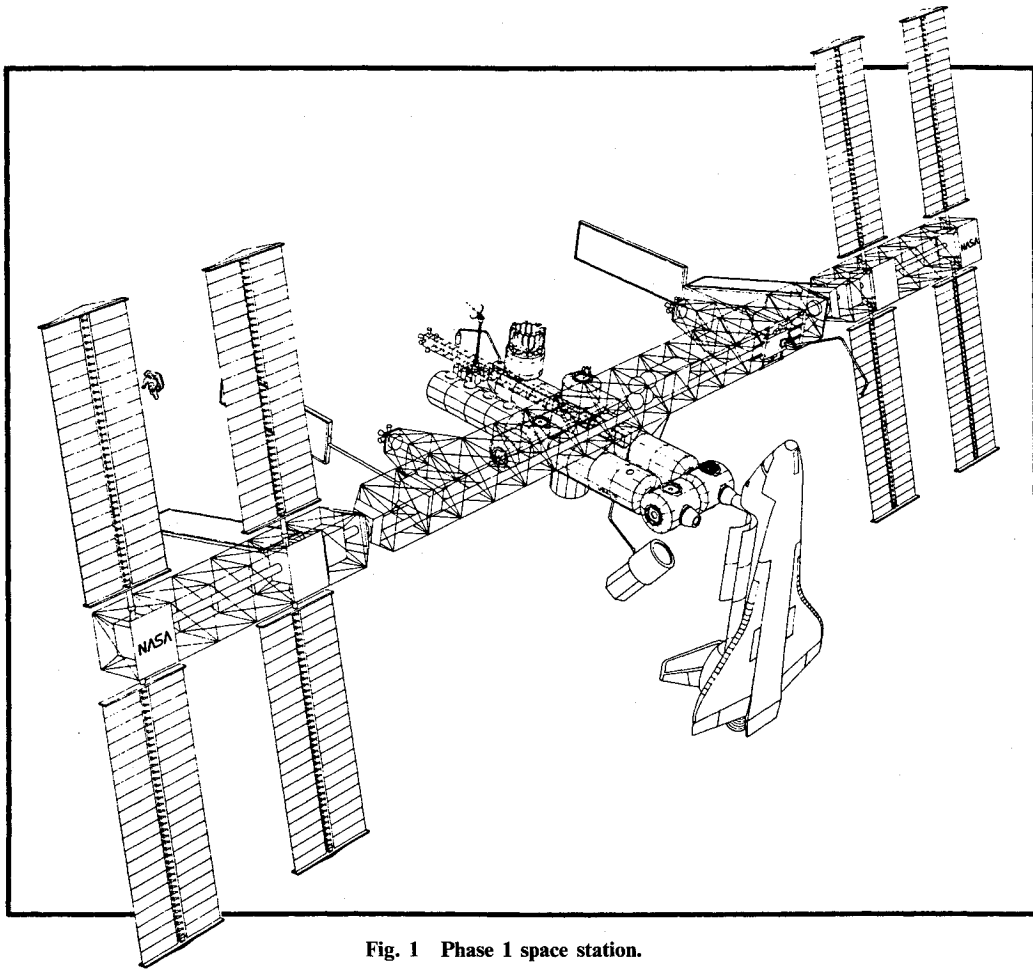


Fig. 1 Phase 1 space station.

where

$$c_1 \triangleq -\sin\theta_2 \cos\theta_3$$

$$c_2 \triangleq \cos\theta_1 \sin\theta_2 \sin\theta_3 + \sin\theta_1 \cos\theta_2$$

$$c_3 \triangleq -\sin\theta_1 \sin\theta_2 \sin\theta_3 + \cos\theta_1 \cos\theta_2$$

Attitude kinematics (2-3-1 body-axis sequence):

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \frac{1}{\cos\theta_3} \begin{bmatrix} \cos\theta_3 & -\cos\theta_1 \sin\theta_3 & \sin\theta_1 \sin\theta_3 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 \cos\theta_3 & \cos\theta_1 \cos\theta_3 \end{bmatrix} \times \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} 0 \\ n \\ 0 \end{bmatrix} \quad (2)$$

CMG momentum:

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (3)$$

where subscripts (1,2,3) refer to the roll, pitch, and yaw control axes, whose origin is fixed at the mass center, with the roll axis in the flight direction, the pitch axis perpendicular to the orbit plane, and the yaw axis toward the Earth;  $(\theta_1, \theta_2, \theta_3)$  are the roll, pitch, and yaw Euler angles of the control (body)

axes with respect to LVLH axes, which rotate with the orbital angular velocity,  $n$ ;  $(\omega_1, \omega_2, \omega_3)$  are the body-axis components of the absolute angular velocity of the station;  $(I_{11}, I_{22}, I_{33})$  are the moments of inertia;  $I_{ij}$  ( $i \neq j$ ) are the products of inertia;  $(h_1, h_2, h_3)$  are the body-axis components of the CMG momentum;  $(u_1, u_2, u_3)$  are the body-axis components of the control torque caused by CMG momentum change;  $(w_1, w_2, w_3)$  are the body-axis components of the external disturbance torque; and  $n$  is the orbital rate of 0.0011 rad/s.

For small attitude deviations from LVLH orientation, the linearized equations of motion can be written as

Space station dynamics:

$$\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = n \begin{bmatrix} I_{31} & 2I_{32} & I_{33} - I_{22} \\ -I_{32} & 0 & I_{12} \\ I_{22} - I_{11} & -2I_{12} & -I_{13} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} + 3n^2 \begin{bmatrix} I_{33} - I_{22} & I_{21} & 0 \\ I_{12} & I_{33} - I_{11} & 0 \\ -I_{13} & -I_{23} & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + n^2 \begin{bmatrix} -2I_{23} \\ 3I_{13} \\ -I_{12} \end{bmatrix} + \begin{bmatrix} -u_1 + w_1 \\ -u_2 + w_2 \\ -u_3 + w_3 \end{bmatrix} \quad (4)$$

Attitude kinematics:

$$\dot{\theta}_1 - n\theta_3 = \omega_1 \quad (5a)$$

$$\dot{\theta}_2 - n = \omega_2 \quad (5b)$$

$$\dot{\theta}_3 + n\theta_1 = \omega_3 \quad (5c)$$

CMG momentum:

$$\dot{h}_1 - nh_3 = u_1 \quad (6a)$$

$$\dot{h}_2 = u_2 \quad (6b)$$

$$\dot{h}_3 + nh_1 = u_3 \quad (6c)$$

By eliminating  $(\omega_1, \omega_2, \omega_3)$  from Eqs. (4) and (5), the linearized equations of motion can also be written as

$$\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} = n \begin{bmatrix} 0, & 2I_{32}, & I_{11} - I_{22} + I_{33} \\ -2I_{32}, & 0, & 2I_{12} \\ -I_{11} + I_{22} - I_{33}, & -2I_{12}, & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \\ + n^2 \begin{bmatrix} 4(I_{33} - I_{22}), & 3I_{21}, & -I_{31} \\ 4I_{12}, & 3(I_{33} - I_{11}), & I_{32} \\ -4I_{13}, & -3I_{23}, & I_{11} - I_{22} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \\ + n^2 \begin{bmatrix} -4I_{23} \\ 3I_{13} \\ I_{12} \end{bmatrix} + \begin{bmatrix} -u_1 + w_1 \\ -u_2 + w_2 \\ -u_3 + w_3 \end{bmatrix} \quad (7)$$

Note that the products of inertia cause three-axis coupling as well as a bias torque in each axis. Early flight configurations during station assembly may have significant misalignment of the control axes with the principal axes. In such cases, the preceding linear, three-axis coupled equations of motion will be very useful for linear control design. It is by now evident that it is very straightforward to derive Eq. (7); however, to the authors' knowledge, this does not appear to have been done in the literature.

Fortunately, most practical situations of interest with small products of inertia permit further simplification in such a way that pitch motion is uncoupled from roll/yaw motion. Otherwise, Eq. (7) should be used for three-axis coupled stability analysis and control design. For the case where the control axes are nearly aligned with the principal axes ( $I_1 \triangleq I_{11}$ ,  $I_2 \triangleq I_{22}$ , and  $I_3 \triangleq I_{33}$ ), Eqs. (4) becomes

$$I_1 \dot{\omega}_1 + n(I_2 - I_3)\omega_3 + 3n^2(I_2 - I_3)\theta_1 = -u_1 + w_1 \quad (8a)$$

$$I_2 \dot{\omega}_2 + 3n^2(I_1 - I_3)\theta_2 = -u_2 + w_2 \quad (8b)$$

$$I_3 \dot{\omega}_3 - n(I_2 - I_1)\omega_1 = -u_3 + w_3 \quad (8c)$$

Finally, the linear equations for  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  become

$$I_1 \ddot{\theta}_1 + 4n^2(I_2 - I_3)\theta_1 - n(I_1 - I_2 + I_3)\theta_3 = -u_1 + w_1 \quad (9a)$$

$$I_2 \ddot{\theta}_2 + 3n^2(I_1 - I_3)\theta_2 = -u_2 + w_2 \quad (9b)$$

$$I_3 \ddot{\theta}_3 + n^2(I_2 - I_1)\theta_3 + n(I_1 - I_2 + I_3)\theta_1 = -u_3 + w_3 \quad (9c)$$

These are the important equations used for the study of passive or active gravity-gradient stabilization of Earth-pointing satellites (e.g., see Refs. 16 and 18). Since pitch motion is uncoupled from roll/yaw motion, pitch control is often analyzed separately from coupled roll/yaw control.

However, certain configurations of the space station (e.g., assembly flight 3 of Table 1) may need a large pitch TEA because of the small gravity-gradient torque available in the pitch axis. In such cases, Eqs. (1) and (2) with small roll/yaw attitude errors and small products of inertia become<sup>25,26</sup>:

$$I_1 \ddot{\theta}_1 + (1 + 3 \cos^2 \theta_2)n^2(I_2 - I_3)\theta_1 - n(I_1 - I_2 + I_3)\theta_3 \\ + 3(I_2 - I_3)n^2(\sin \theta_2 \cos \theta_2)\theta_3 = -u_1 + w_1 \quad (10a)$$

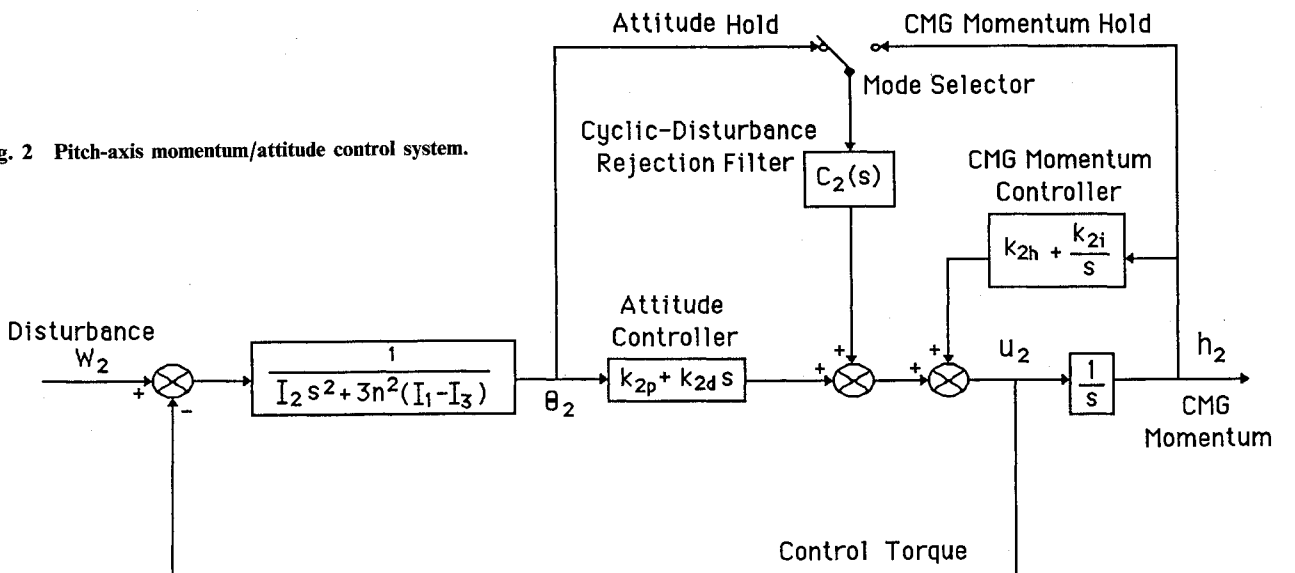
$$I_2 \ddot{\theta}_2 + 3n^2(I_1 - I_3) \sin \theta_2 \cos \theta_2 = -u_2 + w_2 \quad (10b)$$

$$I_3 \ddot{\theta}_3 + (1 + 3 \sin^2 \theta_2)n^2(I_2 - I_1)\theta_3 + n(I_1 - I_2 + I_3)\theta_1 \\ + 3(I_2 - I_1)n^2(\sin \theta_2 \cos \theta_2)\theta_1 = -u_3 + w_3 \quad (10c)$$

Table 1 Space station parameters

Parameters	Assembly flight 3	Phase 1
Inertia (slug-ft <sup>2</sup> ):		
$I_{11}$	23.22E6	50.28E6
$I_{22}$	1.30E6	10.80E6
$I_{33}$	23.23E6	58.57E6
$I_{12}$	-0.023E6	-0.39E6
$I_{13}$	0.477E6	0.16E6
$I_{23}$	-0.011E6	0.16E6
Aerodynamic torque (ft-lb) for phase 1:		
$w_1$	$1 + \sin(nt) \times 0.5 \sin(2nt)$	
$w_2$	$4 + 2 \sin(nt) + 0.5 \sin(2nt)$	
$w_3$	$1 + \sin t(n) + 0.5 \sin(2nt)$	

Fig. 2 Pitch-axis momentum/attitude control system.



It is evident that roll/yaw motion is now affected by pitch motion. If pitch attitude is held constant with respect to the pitch TEA, the roll/yaw equations can be considered time invariant. Thus, these equations also will be useful in roll/yaw controller design for the space stations with large pitch TEA.

Inertia matrices of the phase 1 space station as well as assembly no. 3 are listed in Table 1. In this paper, we are concerned primarily with the phase 1 configuration. The uncontrolled space station with such inertial properties is in an unstable equilibrium when  $\theta_i = 0$  ( $i = 1, 2, 3$ ). Also included are expected aerodynamic disturbances that are modeled as bias plus cyclic terms in the body-fixed control axes, as follows:

$$w(t) = \text{Bias} + A_n \sin(nt + \phi_n) + A_{2n} \sin(2nt + \phi_{2n}) \quad (11)$$

The cyclic component at the orbital rate is caused by Earth's diurnal bulge, whereas the cyclic torque at twice the orbital rate is caused by the rotating solar panels. The magnitudes and phases of aerodynamic torque in each axis are assumed unknown for control design.

### Pitch-Axis Controller Synthesis

Equation (9b), which is linearized and uncoupled from the roll/yaw equations, is used as the basis for pitch control analysis and design. A block diagram of the pitch-axis momentum/attitude control loop is shown in Fig. 2. Two control schemes are presented here: one without the indicated cyclic-disturbance rejection filter,  $C_2(s)$ , and one including the filter.

#### Pitch Control without Cyclic-Disturbance Rejection

The pitch-axis controller consists of a single control input,  $u_2$ , and four states,  $\theta_2$ ,  $\dot{\theta}_2$ ,  $h_2$ ,  $\int h_2$ . The pitch control logic is then given by

$$u_2 = K_{2P}\theta_2 + K_{2D}\dot{\theta}_2 + K_{2H}h_2 + K_{2I}\int h_2 \quad (12)$$

where the pitch-axis CMG momentum and its integral are included to prevent momentum buildup.

Various methods for the selection of the four gains can be employed. For example, one can place the closed-loop eigenvalues at any desired location by using the pole placement technique. The practical problem with this approach is that it is not always clear where to place the eigenvalues for satisfactory performance and robustness.

Pitch controller synthesis by iterative successive loop closures is possible but quite tedious. The pitch controller also can be synthesized using an "equivalent" compensator. This approach provides physical insight into the proper tradeoff between the attitude control and the momentum management. Combining Eqs. (6b) and (12) to eliminate  $h_2$  gives an equivalent compensator:

$$u_2 = \frac{s^2(K_{2P} + sK_{2D})}{s^2 - K_{2H}s - K_{2I}} \theta_2 \quad (13)$$

Hence, the integrated momentum/attitude controller can be interpreted as a second-order compensator with four parameters. In the complex  $s$ -plane, the momentum controller consists of double zero at the origin and complex poles. The attitude controller consists of one zero on the real axis, which is a conventional proportional-derivative controller. An unstable compensator is needed to stabilize the unstable system. It can easily be shown that  $h_2(s)/w_2(s)$  of the closed-loop system has a zero at  $s = 0$ . Thus,  $h_2(t)$  has a zero steady-state value for a constant disturbance.

LQR synthesis of the pitch control with full state feedback in  $u_2$  can be accomplished quickly with a computer code. The LQR technique makes use of a quadratic performance index to synthesize state feedback gains that minimize this index. It

is especially useful for multivariable systems; all loops are closed simultaneously instead of successively, as in classical frequency-domain methods. However, the problem with this approach is that the proper selection of the weighting matrices is not obvious. It is not always possible to predict the effects of given weighting matrices on the closed-loop behavior.

Hence, a practical approach to the pitch-axis controller design (and the roll/yaw controller design of the next section) would be to find the control gains and closed-loop poles that result from a wide range of weighting matrices, and simulate the corresponding closed-loop system. The gain matrix that produces the closed-loop responses satisfying the various requirements becomes the final selection.

The uncontrolled phase 1 space station has an unstable pitch mode with poles at  $s = \pm 1.5n$  and a momentum mode with a double pole at  $s = 0$  (one from integral feedback). After iterative use of an LQR synthesis code, available in the CTRL-C software, we select the pitch-axis control gains as listed in Table 2. The corresponding closed-loop eigenvalues are as follows:

$$-1.6n, \quad -1.0n, \quad (-1.5 \pm 1.5j)n$$

The response of the closed-loop system to the pitch disturbance listed in Table 1, with initial conditions of  $\theta_2(0) = 1$  deg and  $\dot{\theta}_2(0) = 0.001$  deg/s is shown in Fig. 3. The transient responses are satisfactory, whereas the cyclic aerodynamic torque causes the periodic responses of both the pitch-axis CMG momentum and pitch attitude. The CMG momentum  $h_2$  is bounded with a zero mean value, whereas  $\theta_2$  is oscillating with respect to a 7.5 deg pitch TEA. The CMG momentum peak and torque demand are both well below the allowable limits of about 20,000 ft-lb-s and 150 ft-lb, respectively.

#### Pitch Control with Cyclic-Disturbance Rejection

Depending on the circumstances, either pitch attitude or CMG momentum oscillation, caused by the aerodynamic

Table 2 Controller gains without cyclic-disturbance rejection filter for the phase 1 space station without disturbance rejection filters

Gain	Pitch	Roll	Yaw
$K_P$ , ft-lb/rad	$2.448E+2$	$1.113E+3$	$1.173E+3$
$K_D$ , ft-lb/rad/s	$1.465E+5$	$5.161E+5$	$1.283E+6$
$K_H$ , ft-lb/ft-lb-s	$7.523E-3$	$5.935E-3$	$1.540E-2$
$K_I$ , ft-lb/ft-lb-s <sup>2</sup>	$3.546E-6$	$1.105E-5$	$1.990E-6$

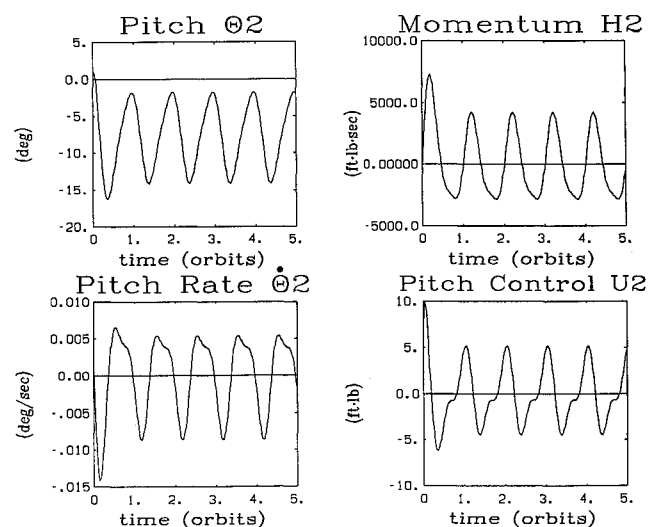


Fig. 3 Pitch-axis response without disturbance rejection filtering.

disturbance torque, may be undesirable. In such cases, a cyclic-disturbance rejection filter can be employed as illustrated in Fig. 2.

The proposed cyclic-disturbance rejection filter,  $C_2(s)$ , is represented as

$$C_2(s) = \frac{N_2(s)}{[s^2 + n^2][s^2 + (2n)^2]} \quad (14)$$

The filter poles will appear in the numerator of the closed-loop transfer function  $\theta_2(s)/w_2(s)$  or  $h_2(s)/w_2(s)$ , depending on mode selection. This results in a disturbance rejection at frequencies of  $n$  and  $2n$  for either  $\theta_2$  or  $h_2$ . The filter numerator,  $N_2(s)$ , should be designed properly to stabilize the overall control system.

For example, the disturbance rejection filters for  $\theta_2$  can be represented as follows:

$$\ddot{\alpha}_2 + (n)^2 \alpha_2 = \theta_2 \quad (15a)$$

$$\ddot{\beta}_2 + (2n)^2 \beta_2 = \theta_2 \quad (15b)$$

where initial conditions for  $\alpha_2$ ,  $\beta_2$ ,  $\dot{\alpha}_2$ , and  $\dot{\beta}_2$  can be selected arbitrarily (usually, zero initial conditions). These filter equations are integrated in the onboard computer.

We now repeat the pitch control synthesis of the previous section, augmenting the pitch-axis equation with preceding filter equations. The pitch control logic, with additional disturbance rejection filter states, can then be expressed as

$$u_2 = K_{2P}\theta_2 + K_{2D}\dot{\theta}_2 + K_{2H}h_2 + K_{2I}\int_0^t h_2 dt + K_{2\alpha}\alpha_2 + K_{2\dot{\alpha}}\dot{\alpha}_2 + K_{2\beta}\beta_2 + K_{2\dot{\beta}}\dot{\beta}_2 \quad (16)$$

The eight gains of Eq. (16) can be determined using either the pole-placement or LQR technique. We use the LQR synthesis technique, and the selected control gains and the corresponding closed-loop eigenvalues are listed in Tables 3 and 4, respectively.

Since the disturbance rejection filter is used for pitch attitude, it can be shown that  $\theta_2(s)/w_2(s)$  has zeros at  $s = \pm nj$  and  $\pm 2nj$ ; hence, an asymptotic disturbance rejection of a cyclic disturbance of frequencies  $n$  and  $2n$  is achieved for the pitch attitude. Transfer function  $h_2(s)/w_2(s)$  also has zeros near  $s = \pm nj$  and  $\pm 2nj$ . This results in minimizing the cyclic peak of the CMG momentum. Closed-loop responses of this controller are shown in Fig. 4. The CMG momentum is bounded with a zero mean value, whereas the pitch attitude is now held constant at a 7.5 deg pitch TEA after two orbits. These responses should be compared to those in Fig. 3, where no disturbance rejection filter was used. It can be seen that the cyclic peak of the CMG momentum is reduced. In other words, the use of disturbance rejection filtering results in minimizing the overall oscillations of attitude and CMG momentum.

**Table 3** Controller gains with cyclic-disturbance rejection filter for the phase 1 space station with disturbance rejection filters

Gain	Pitch	Roll	Yaw
$K_P$ , ft-lb/rad	$3.425E+2$	$7.059E+2$	$9.253E+2$
$K_D$ , ft-lb/rad/s	$1.972E+5$	$4.408E+5$	$4.165E+5$
$K_H$ , ft-lb/ft-lb-s	$1.089E-2$	$4.816E-3$	$1.833E-4$
$K_I$ , ft-lb/ft-lb-s <sup>2</sup>	$3.953E-6$	$-8.228E-7$	$-1.469E-6$
$K_{\alpha}$ , ft-lb/rad	$1.006E-4$	$4.414E-10^a$	$-3.399E-4^b$
$K_{\dot{\alpha}}$ , ft-lb/rad/s	$5.659E-2$	$8.038E-7^b$	$5.496E-3^c$
$K_{\beta}$ , ft-lb/rad	$1.434E-5$	$-3.329E-9^a$	$-4.416E-4^b$
$K_{\dot{\beta}}$ , ft-lb/rad/s	$7.608E-2$	$-4.823E-7^b$	$2.430E-1^b$

<sup>a</sup>In units of ft-lb-rad<sup>2</sup>/ft-lb-s<sup>3</sup>. <sup>b</sup>In units of ft-lb-rad<sup>2</sup>/ft-lb-s<sup>2</sup>. <sup>c</sup>In units of ft-lb-rad<sup>2</sup>/ft-lb-s.

It is also possible to provide cyclic-disturbance rejection for the pitch-axis CMG momentum. In this case, we replace  $\theta_2$  of Eqs. (15) by  $h_2$  and then find proper gains for Eq. (16). The closed-loop response of this case is shown in Fig. 5. The preceding results show that, depending on the specific mission requirements, either pitch attitude or CMG momentum can be held constant by employing the pitch controller illustrated in Fig. 2.

### Roll/Yaw Controller Synthesis

A design procedure similar to that of the pitch-axis design is followed for the roll/yaw controller. Two cases are considered: the first without cyclic-disturbance rejection filters, the second with filters.

#### Roll/Yaw Control without Cyclic-Disturbance Rejection

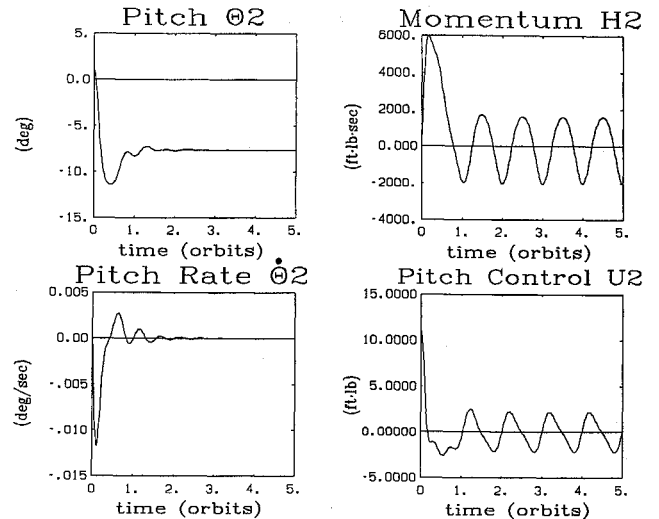
The first case of roll/yaw controller design consists of two inputs,  $u_1$  and  $u_3$ , and eight states, including two integral states for CMG momentum control. The eight states are as follows:

$$\text{Roll axis: } \theta_1 \quad \omega_1 \quad h_1 \quad \int_0^t h_1 dt$$

$$\text{Yaw axis: } \theta_3 \quad \omega_3 \quad h_3 \quad \int_0^t h_3 dt$$

Note that we use  $\omega_1$  and  $\omega_3$ , instead of  $\dot{\theta}_1$  and  $\dot{\theta}_3$ , as state variables.

The uncontrolled phase 1 space station, with the inertial properties listed in Table 1, is unstable in roll/yaw and has the open-loop poles of  $(\pm 1.05 \pm 0.7j)n$ ,  $\pm nj$ , 0, 0. The double pole at  $s=0$  is due to the integral feedback of  $h_1$  and  $h_3$ . Similar to the pitch-axis design, the roll/yaw full-state feedback controller can be designed, iteratively, using the LQR or pole-placement technique. The multi-input characteristics of



**Fig. 4** Pitch-axis response with disturbance rejection filtering for the pitch attitude.

**Table 4** Closed-loop eigenvalues for the phase 1 in units of orbital rate,  $n = 0.0011$  rad/s

	Momentum/attitude		Disturbance filter	
Pitch	-1.0, -1.5	$-1.5 \pm 1.5j$	$-0.3 \pm 1.0j$	$-0.3 \pm 2.0j$
Roll/yaw	-0.23, -0.68	$-0.66 \pm 1.51j$	$-0.23 \pm 0.92j$	$-0.20 \pm 2.02j$
	$-1.02 \pm 0.29j$	$-1.50 \pm 0.84j$	$-0.26 \pm 1.04j$	$-0.62 \pm 2.29j$

the roll/yaw axes, however, provide for the calculation of various gain matrices that yield the same closed-loop eigenvalues. Although the gain matrix can be specified completely by assigning not only the closed-loop eigenvalues but also an allowable set of closed-loop eigenvectors, there are many open issues in the eigensystem assignment approach.<sup>22-24</sup> In our study, we simply use the conventional LQR technique to find a closed-loop system satisfying various requirements.

After iterative use of an LQR synthesis code, we select a  $2 \times 8$  gain matrix with the corresponding closed-loop eigenvalues of  $(-1.05 \pm 0.68j)n$ ,  $(-1.04 \pm 0.72j)n$ ,  $(-1.42 \pm 1.38j)n$ , and  $(-1.42 \pm 1.38j)n$ . This choice of closed-loop eigenvalues is not claimed to be the best.

We now discuss some motivation for developing a decentralized, partial state feedback controller for the space station. The configurations of the space station will be changing as payloads/modules are attached or removed. This drives the need for a control system design that is either insensitive to the configuration changes or able to adapt autonomously. At present, conventional design methods, including gain scheduling or mode switching, appear adequate for most of the space station flight configurations. Some drawbacks of nonadaptive controllers are that the design is often time-consuming, and the controller parameters (e.g., the 16 gains of the preceding roll/yaw controller) must be determined for many operating conditions. Furthermore, they must be stored in the onboard computer.

In order to minimize the computer storage requirement for gain scheduling, a roll/yaw control logic and a reduced number of gains is considered here. One possibility might be to use an LQR synthesis code to find a full-state feedback gain matrix and then omit some of the less important gains. However, this approach does not always guarantee good performance and stability. Hence, we have developed an iterative pole-placement algorithm that is an extension of the Bass-Gura formula for single-input systems. A brief summary of the algorithm for multi-input systems is given in the Appendix.

A decentralized, partial state feedback control logic proposed for the space station is simply

$$u_1 = K_{1P}\theta_1 + K_{1D}\omega_1 + K_{1H}h_1 + K_{1I} \int_0^t h_1 dt \quad (17a)$$

$$u_3 = K_{3P}\theta_3 + K_{3D}\omega_3 + K_{3H}h_3 + K_{3I} \int_0^t h_3 dt \quad (17b)$$

Notice a particular feedback structure wherein each control input uses states that are more closely related to that control input. We select the eight gains by choosing the closed-loop eigenvalues to be the optimal, full-state regulator eigenvalues determined previously. The eight gains obtained using the pole-placement algorithm described in the Appendix are listed in Table 2. A simulation of the closed-loop system with roll/yaw disturbances of Table 1 shows that the transient responses are satisfactory and that the steady-state responses are cyclic and remain within specified limits. The decentralized controller with eight gains compares quite favorably with the full-state feedback controller with 16 gains.

#### Roll/Yaw Control with Cyclic-Disturbance Rejection

For the minimization of the steady-state oscillation of roll/yaw attitude and CMG momentum, cyclic-disturbance rejection filtering for a multivariable system is investigated.

To clarify some multivariable control issues related to transmission zeros,<sup>20</sup> let us consider the open-loop transfer-function matrix from control inputs to roll/yaw attitude and CMG momentum. By combining Eqs. (6a), (6c), (9a), and (9c), we can write the transfer-function matrix as:

$$\begin{bmatrix} h_1(s) \\ h_3(s) \end{bmatrix} = \frac{1}{s^2 + n^2} \begin{bmatrix} s & n \\ -n & s \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_3(s) \end{bmatrix} \quad (19)$$

where

$$\Delta = I_1 I_3 (s^2 + n^2) [s^4 + (1 + 3k_1 + k_1 k_3) n^2 s^2 + 4k_1 k_3 n^4]$$

$$k_1 = (I_2 - I_3)/I_1$$

$$k_3 = (I_2 - I_1)/I_3$$

It is apparent that  $\pm nj$  are transmission zeros of Eq. (18). We also notice from Eq. (19) that resonance of CMG momentum can occur for sinusoidal control inputs of frequency  $n$ . In other words, a cyclic disturbance at an orbital rate cannot be rejected for both roll and yaw attitude. However, in this case, where we use the CMG momentum dynamics represented as Eqs. (6), it is not evident why a cyclic disturbance at an orbital rate can be rejected for the yaw attitude and not for the roll attitude. In order to prove such inherent characteristics

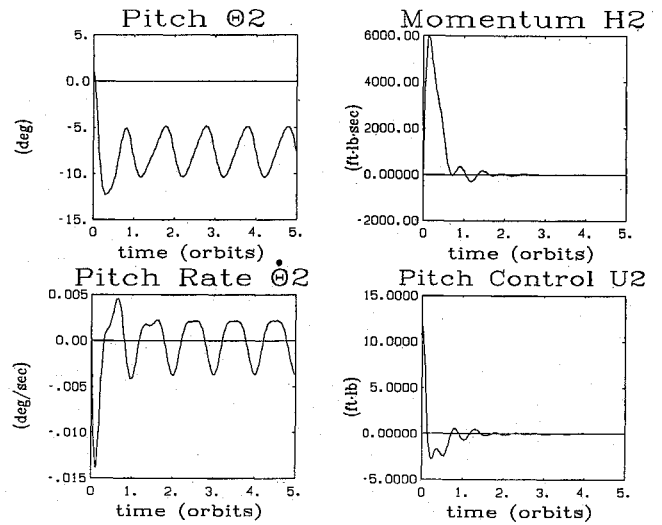


Fig. 5 Pitch-axis response with disturbance rejection filtering for the pitch-axis CMG momentum.

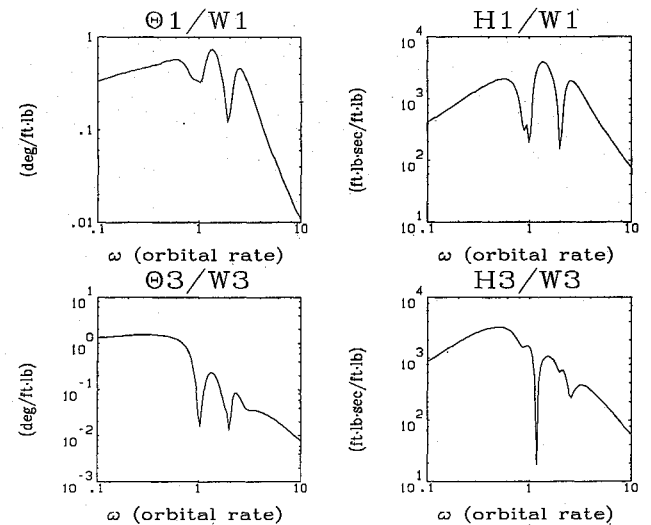


Fig. 6 Closed-loop frequency response for the roll/yaw axes with disturbance rejection filtering for the yaw attitude and roll-axis CMG momentum.

$$\begin{bmatrix} \theta_1(s) \\ \theta_3(s) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -[I_3 s^2 + (I_2 - I_1) n^2] (s^2 + n^2) & -[(I_1 - I_2 + I_3) n s] (s^2 + n^2) \\ [(I_1 - I_2 + I_3) n s] (s^2 + n^2) & -[I_1 s^2 + 4(I_2 - I_3) n^2] (s^2 + n^2) \end{bmatrix} \begin{bmatrix} u_1(s) - w_1(s) \\ u_3(s) - w_3(s) \end{bmatrix} \quad (18)$$

of roll/yaw dynamics, the following CMG momentum dynamics with a proper modification of Eqs. (9) are investigated

$$\dot{h}_1 = u_1 \quad (20a)$$

$$\dot{h}_3 = u_3 \quad (20b)$$

For this case, the following transfer-function matrix can be obtained:

$$\begin{bmatrix} \theta_1(s) \\ \theta_3(s) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -I_3 s^2 (s^2 + n^2), & (I_2 - I_1) n s (s^2 + n^2) \\ n s (I_3 - I_2) [s^2 + (2n)^2], & -s^2 [I_1 s^2 + (I_1 + 3I_2 - 3I_3) n^2] \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_3(s) \end{bmatrix} + \frac{s^2}{\Delta} \begin{bmatrix} I_3 s^2 + (I_2 - I_1) n^2, & (I_1 - I_2 + I_3) n s \\ -(I_1 - I_2 + I_3) n s, & I_1 s^2 + 4(I_2 - I_3) n^2 \end{bmatrix} \begin{bmatrix} w_1(s) \\ w_3(s) \end{bmatrix} \quad (21)$$

$$\begin{bmatrix} h_1(s) \\ h_3(s) \end{bmatrix} = \frac{1}{s} \begin{bmatrix} 1, & 0 \\ 0, & 1 \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_3(s) \end{bmatrix} \quad (22)$$

where

$$\begin{aligned} \Delta &= I_1 I_3 s^2 [s^4 + (1 + 3k_1 + k_1 k_3) n^2 s^2 + 4k_1 k_3 n^4] \\ k_1 &= (I_2 - I_3) / I_1 \\ k_3 &= (I_2 - I_1) / I_3 \end{aligned}$$

It can be shown that  $\pm nj$  are transmission zeros of the transfer-function matrix from  $(u_1, u_3)$  to  $(\theta_1, \theta_3)$ . The zero at  $s = \pm nj$  appears in both the  $\theta_1(s)/u_1(s)$  and  $\theta_1(s)/u_3(s)$  transfer functions, but not in yaw attitude channels. Therefore, a cyclic-disturbance rejection at the orbital rate is not possible for roll attitude, whereas it is possible for yaw attitude. Furthermore, it can be shown that  $\pm nj$  are not transmission zeros of the transfer-function matrix from  $(u_1, u_3)$  to  $(h_1, \theta_3)$ . Based on these inherent properties of the roll/yaw dynamics, a cyclic-disturbance rejection for the roll-axis CMG momentum and yaw attitude is considered.

The roll/yaw cyclic-disturbance rejection filters proposed for the space station are as follows:

$$\ddot{\alpha}_1 + (n)^2 \alpha_1 = h_1 \quad (23a)$$

$$\ddot{\beta}_1 + (2n)^2 \beta_1 = h_1 \quad (23b)$$

$$\ddot{\alpha}_3 + (n)^2 \alpha_3 = \theta_3 \quad (23c)$$

$$\ddot{\beta}_3 + (2n)^2 \beta_3 = \theta_3 \quad (23d)$$

By using a LQR synthesis code, we can determine a  $2 \times 16$  gain matrix for the roll/yaw equations augmented by the preceding filter equations. However, we consider here a decentralized, partial state feedback controller in order to minimize the computer storage requirement for gain scheduling.

A decentralized control logic with a cyclic-disturbance accommodation is given by

$$\begin{aligned} u_1 &= K_{1p} \theta_1 + K_{1D} \dot{\theta}_1 + K_{1H} h_1 + K_{1I} \int_0^t h_1 dt \\ &\quad + K_{1\alpha} \alpha_1 + K_{1\dot{\alpha}} \dot{\alpha}_1 + K_{1\beta} \beta_1 + K_{1\dot{\beta}} \dot{\beta}_1 \end{aligned} \quad (24a)$$

$$\begin{aligned} u_3 &= K_{3p} \theta_3 + K_{3D} \dot{\theta}_3 + K_{3H} h_3 + K_{3I} \int_0^t h_3 dt \\ &\quad + K_{3\alpha} \alpha_3 + K_{3\dot{\alpha}} \dot{\alpha}_3 + K_{3\beta} \beta_3 + K_{3\dot{\beta}} \dot{\beta}_3 \end{aligned} \quad (24b)$$

Proper closed-loop eigenvalues are first selected from LQR design iterations, and the 16 gains of Eqs. (24) are then determined by using the pole-placement algorithm derived in the Appendix. These gains and closed-loop eigenvalues are listed in Tables 3 and 4, respectively. It is emphasized that the decentralized control is not necessary if a state feedback controller with 32 gains can be easily implemented without any onboard computer storage problem for many different

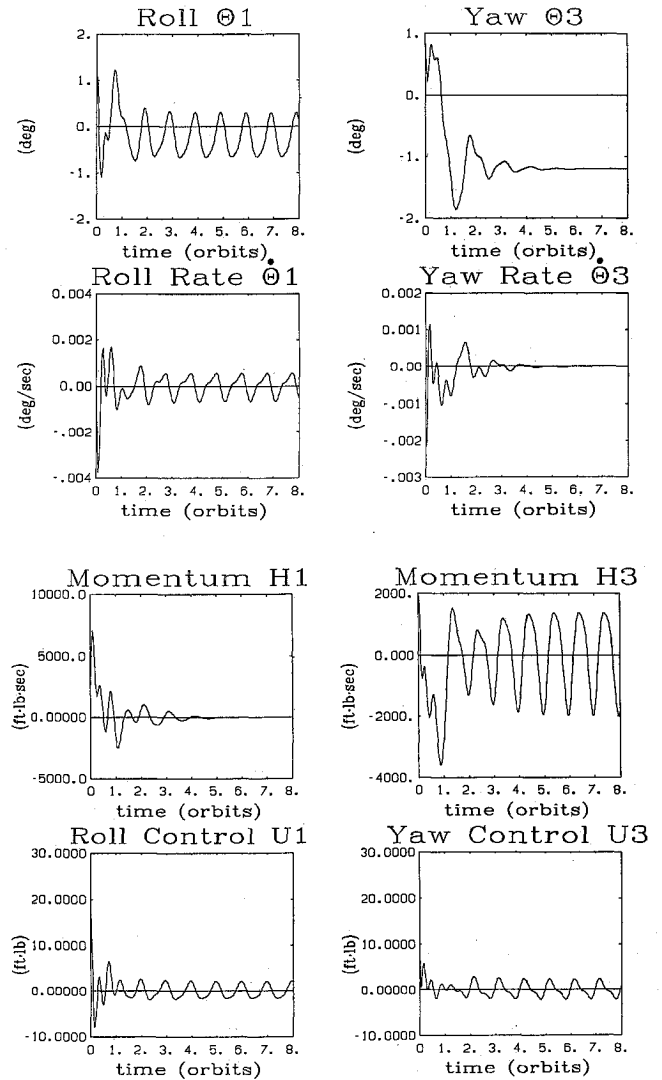


Fig. 7 Roll and yaw responses.

operating conditions. The decentralized controller is proposed here as an additional capability of our control system for the space station.

Bode plots of the closed-loop transfer functions are shown in Fig. 6. Both  $h_1(s)/w_1(s)$  and  $\theta_3(s)/w_3(s)$  have zeros at  $s = \pm nj$  and  $\pm 2nj$ ; hence, an asymptotic disturbance rejection for the roll-axis CMG momentum and yaw attitude is achieved for cyclic disturbances of frequencies  $n$  and  $2n$ . It can be shown that  $\pm nj$  are blocking zeros of the closed-loop transfer-function matrix from  $(w_1, w_3)$  to  $(h_1, \theta_3)$ . Also,  $\theta_1(s)/w_1(s)$  and  $h_3(s)/w_3(s)$  have zeros near  $s = \pm nj$  and  $\pm 2nj$ , which results in minimizing the cyclic peak of the roll attitude and yaw-axis

CMG momentum. The responses to the roll/yaw disturbances listed in Table 1, with initial conditions of  $\theta_1(0) = \theta_3(0) = 1$  deg and  $\omega_1(0) = \omega_3(0) = 0.001$  deg/s, are shown in Fig. 7. The roll-axis CMG momentum approaches a zero steady-state value, whereas roll attitude oscillates at the orbital rate. The yaw attitude approaches a constant steady-state value, whereas the yaw-axis CMG momentum oscillates at the orbital rate. As a result, the overall attitude and CMG momentum oscillations are minimized. In fact, the responses of the decentralized controller with 16 gains compare quite favorably with those of a full-state feedback controller with 32 gains.

From simulations, it was verified that the overall closed-loop system, with the feedback gains of Table 3, is stable for  $\pm 30\%$  inertia matrix variations, and that it has more than  $\pm 10$  dB and 45 deg stability margins for each loop.

### Conclusions

A new approach to the space station momentum management and attitude control is presented. The proposed control algorithm provides a proper cyclic-disturbance accommodation. We have shown that either pitch attitude or pitch-axis CMG momentum, depending on the specific mission requirements, can be held constant. We have also found an inherent physical property of the coupled roll/yaw dynamics in terms of a transmission zero of a multivariable system. A decentralized, partial state feedback controller, which requires minimum computer storage requirement for gain scheduling, was also developed.

### Appendix:

#### Pole-Placement Algorithm for Partial State Feedback

A pole-placement algorithm for the decentralized, partial-state feedback control design is presented. This iterative algorithm for a multi-input system is an extension of the Bass-Gura formula for a single-input system.

Consider a linear, time-invariant system with a single input described by

$$\dot{x} = Ax + bu$$

$$u = -kx$$

where  $x$  is the  $n$ -dimensional state vector,  $u$  the scalar input,  $A$  an  $n \times n$  matrix,  $b$  an  $n \times 1$  column vector, and  $k$  a  $1 \times n$  row vector.

The Bass-Gura formula for determining the feedback gain,  $k$ , of a controllable system is then given by

$$a_c - a_o = kCT$$

where

$$\begin{aligned} \Delta(s) &= \text{characteristic polynomial,} \\ &= s^n + a_1s^{n-1} + \dots + a_{n-1}s + a_n \\ a_o &= [a_{1o}, a_{2o}, \dots, a_{no}] \text{ for the open-loop system} \\ a_c &= [a_{1c}, a_{2c}, \dots, a_{nc}] \text{ for the closed-loop system} \\ C &= \text{controllability matrix, } = [b, Ab, \dots, A^{n-1}b] \\ T &= \text{upper-Toeplitz matrix whose first row is} \\ &[1, a_1, \dots, a_{n-1}] \end{aligned}$$

Modification can be made to the Bass-Gura formula for a multi-input system described by

$$\dot{x} = Ax + Bu, \quad u = -Kx$$

where  $u$  is the  $m$ -dimensional input,  $B$  is an  $n \times m$  matrix, and  $K$  is an  $m \times n$  gain matrix.

Let  $b_i$  be the  $i$ th column of  $B$ ; i.e.,  $B = [b_1, b_2, \dots, b_m]$ . Also let  $k_i$  be the  $i$ th row of  $K$ . Defining  $A^{(j)}$  to be the closed-loop

system matrix at the  $j$ th iteration, the algorithm now can be described as

$$\begin{aligned} \dot{x} &= A^{(j)}x + \sum_{i=1}^m b_i du_i^{(j+1)} \\ u_i^{(j)} &= u_i^{(j-1)} + du_i^{(j)} \end{aligned}$$

where  $du_i^{(j+1)}$  is used in the  $(j+1)$ th iteration to improve the convergence to the desired eigenvalues. The incremental feedback gain is related to  $du_i^{(j)}$  as

$$\begin{aligned} du_i^{(j)} &= -dk_i^{(j)}x \\ k_i^{(j)} &= k_i^{(j-1)} + dk_i^{(j)} \end{aligned}$$

Small changes in the feedback gains are related to the small changes in the characteristic polynomial coefficients as

$$da^{(j)} = [dk_1^{(j)}C_1^{(j)} + \dots + dk_m^{(j)}C_m^{(j)}]T^{(j)}$$

where

$$\begin{aligned} C_i^{(j)} &= [b_i, A^{(j)}b_i, \dots, (A^{(j)})^{n-1}b_i] \quad (i = 1, \dots, m) \\ T^{(j)} &= \text{upper-Toeplitz matrix with the first-row,} \\ &[1, a_1^{(j)}, \dots, a_{n-1}^{(j)}] \end{aligned}$$

The preceding iterative algorithm can be considered a general pole placement technique for multi-input systems. For the full-state feedback problem, the choice of the gains is not unique. There are some methods for the unique selection of the gains, such as eigensystem assignment approaches (see, for example, Refs. 22–24). In this paper, we consider the case where the number of total feedback gains are specified by the number of states. Then a partial controllability matrix is formed from the selected rows of  $C^{(j)}$  matrices by replacing the unselected gain components with zeros. This approach is valid only when the system is controllable by the selected gain components, i.e., when the partial controllability matrix has the rank  $n$ . The space station example is controllable with the decentralized (partial) state feedback. The final gain set is found, after approximately 30 iterations, with gains from the LQR design used as the initial gain values.

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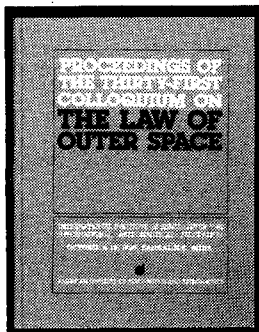
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